NAG Toolbox for MATLAB

f07me

1 Purpose

f07me solves a real symmetric indefinite system of linear equations with multiple right-hand sides,

$$AX = B,$$

where A has been factorized by f07md.

2 Syntax

```
[b, info] = f07me(uplo, a, ipiv, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

f07me is used to solve a real symmetric indefinite system of linear equations $AX = B$, this function must be preceded by a call to f07md which computes the Bunch–Kaufman factorization of A.

If **uplo** = 'U', $A = PUDU^{\mathrm{T}}P^{\mathrm{T}}$, where P is a permutation matrix, U is an upper triangular matrix and D is a symmetric block diagonal matrix with 1 by 1 and 2 by 2 blocks; the solution X is computed by solving $PUDY = B$ and then $U^{\mathrm{T}}P^{\mathrm{T}}X = Y$.

If **uplo** = 'L', $A = PLDL^{\mathrm{T}}P^{\mathrm{T}}$, where L is a lower triangular matrix; the solution X is computed by solving $PLDY = B$ and then $L^{\mathrm{T}}P^{\mathrm{T}}X = Y$.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo – string**

Indicates how A has been factorized.

uplo = 'U'

$A = PUDU^{\mathrm{T}}P^{\mathrm{T}}$, where U is upper triangular.

uplo = 'L'

$A = PLDL^{\mathrm{T}}P^{\mathrm{T}}$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **a(lda,$*$) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

Details of the factorization of A, as returned by f07md.

3: **ipiv**$(*)$ **– int32 array**

Note: the dimension of the array **ipiv** must be at least $\max(1, \mathbf{n})$.

Details of the interchanges and the block structure of D, as returned by f07md.

4: **b**(**ldb**,$*$) **– double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **a** The dimension of the array **ipiv**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p – int32 scalar**

Default: The second dimension of the array **b**.

r, the number of right-hand sides.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldb

5.4 Output Parameters

1: **b**(**ldb**,$*$) **– double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r solution matrix X.

2: **info – int32 scalar**

$\mathbf{info} = 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

$\mathbf{info} = -i$

If $\mathbf{info} = -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **nrhs_p**, 4: **a**, 5: **lda**, 6: **ipiv**, 7: **b**, 8: **ldb**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A+E)x=b$, where

if **uplo** = 'U', $|E| \leq c(n)\epsilon P|U||D|\left|U^{\mathrm{T}}\right|P^{\mathrm{T}}$;
if **uplo** = 'L', $|E| \leq c(n)\epsilon P|L||D|\left|L^{\mathrm{T}}\right|P^{\mathrm{T}}$,

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x-\hat{x}\|_\infty}{\|x\|_\infty} \leq c(n)\,\mathrm{cond}(A,x)\epsilon$$

where $\mathrm{cond}(A,x) = \left\|\left|A^{-1}\right||A||x|\right\|_\infty / \|x\|_\infty \leq \mathrm{cond}(A) = \left\|\left|A^{-1}\right||A|\right\|_\infty \leq \kappa_\infty(A)$.

Note that $\mathrm{cond}(A,x)$ can be much smaller than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling f07mh, and an estimate for $\kappa_\infty(A)$ $(=\kappa_1(A))$ can be obtained by calling f07mg.

8 Further Comments

The total number of floating-point operations is approximately $2n^2r$.

This function may be followed by a call to f07mh to refine the solution and return an error estimate.

The complex analogues of this function are f07ms for Hermitian matrices and f07ns for symmetric matrices.

9 Example

```
uplo = 'L';
a = [2.07, 0, 0, 0;
     3.87, -0.21, 0, 0;
     4.2, 1.87, 1.15, 0;
     -1.15, 0.63, 2.06, -1.81];
b = [-9.5, 27.85;
     -8.38, 9.9;
     -6.07, 19.25;
     -0.96, 3.93];
[a, ipiv, info] = f07md(uplo, a);
[bOut, info] = f07me(uplo, a, ipiv, b)
```

```
bOut =
   -4.0000    1.0000
   -1.0000    4.0000
    2.0000    3.0000
    5.0000    2.0000
info =
           0
```